Generic absoluteness and resurrection axioms

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Generic absoluteness over a theory T containing ZFC is the phenomena by which the truth value of mathematical statements of a certain logical complexity is invariant with respect to appropriate types of forcing which preserve T. This topic has been studied since the introduction of forcing in the late '60, and is motivated by the broad success that the method of forcing reported on consistency results.

These kind of results for a theory T provide a mean to restrict the independence phenomena dating back to Gödel's incompleteness theorems, and can be used to turn the consistency proofs of certain first order statements ϕ into actual derivations (in first order calculus) of ϕ from T.

Classical results in this topic are due to Shoenfield (Δ^1_3-generic absoluteness), Cohen ($\Sigma_1(H_{\omega_1})$-generic absoluteness), Woodin (full generic aboluteness for second order number theory with respect to all forcings under large cardinals) and recently Viale (full generic absoluteness for a large fragment of third order number theory with respect to SSP forcings under large cardinals and strong forcing axioms).

Resurrection axioms were recently introduced by Hamkins and Johnstone as an alternative form of forcing axiom. We introduce the iterated resurrection axioms $\mathrm{RA}_\alpha(\Gamma)$ as α ranges among the ordinals and Γ varies among various classes of forcing notions. Our main results (obtained jointly with Viale) are the following:

Theorem. *If* $\mathrm{RA}_\omega(\Gamma)$ *holds and* $\mathbb{B} \in \Gamma$ *forces* $\mathrm{RA}_\omega(\Gamma)$*, then* $H_\mathfrak{c}^V \prec H_\mathfrak{c}^{V^\mathbb{B}}$ *(where* $\mathfrak{c} = 2^{\aleph_0}$ *is the continuum as computed in the corresponding models).*

Hence a statement $\phi^{H_\mathfrak{c}}$ regarding the structure $H_\mathfrak{c}$ is first order derivable in the theory $T = \mathrm{ZFC} + \mathrm{RA}_\omega(\Gamma)$ whenever T proves its consistency together with T by means of a forcing in Γ.

Theorem. $\mathrm{RA}_\alpha(\Gamma)$ *is consistent relative to the existence of a Mahlo cardinal for the following classes of posets: all, ccc, axiom-A, proper, semiproper. For* $\Gamma = \mathsf{SSP}$ *is consistent relative to the existence of a stationary limit of supercompact cardinals.*

We can also formulate the stronger *boldface resurrection axiom* in an iterated form $\mathrm{R\underset{\sim}{A}}_\alpha(\Gamma)$, and give an exact bound for its consistency strength.

Theorem. $\mathrm{R\underset{\sim}{A}}_\alpha(\Gamma)$ *is equiconsistent with the existence of an* α*-superstrongly unfoldable cardinal for the following classes of posets: all, ccc, axiom-A, proper, semiproper.*

Where α-superstrongly unfoldable cardinals lie strictly below $0^\sharp$ and are compatible with $V = L$. Though, at present from this strengthened axiom we are not able to prove a stronger form of generic absoluteness.

In the talk we shall motivate the foundational role played by generic absoluteness, sketch a proof of some of our results, compare them to the current literature in the field and give a brief account on the open questions still standing.